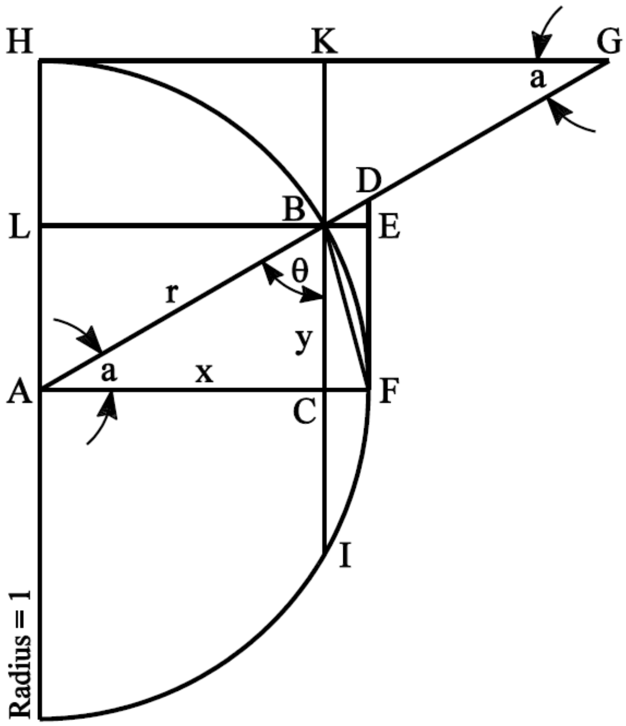
Appendix C
Basic Trigonometry

A. Unit Circle and Formulas



$Radius = 1$
$AF = AB = AH = 1$

$a = angle\ BAC$
$\theta = angle\ ABC$

sine $a = BC = 1/\csc a$
cosine $a = AC = 1/\sec a$
tangent $a = DF = 1/\cot a$
cotangent $a = GH = 1/\tan a$
secant $a = AD = 1/\cos a$
cosecant $a = AC = 1/\sin a$
versine $a = BE = CF = 1 - \cos a$
coversine $a = BK = HL = 1 - \sin a$
exsecant $a = BD = \sec a - 1$
coexsecant $a = BG = \csc a - 1$
haversine a = ½ vers a
chord $a = BF$
chord $2a = 2BC = 2\sin a$

$Hypotenuse\ of\ a = AB = r; Adjacent\ Side = AC = x; Opposite\ Side = BC = y$

$\sin\ a = y/r = \cos\ \theta$
$\cos\ a = x/r = \sin\ \theta$
$tan\ a = y/x = \cot\ \theta$
$\cot\ a = x/y = \tan\ \theta$
$\sec\ a = r/x = csc\ \theta$
$\csc\ a = r/y = \sec\ \theta$

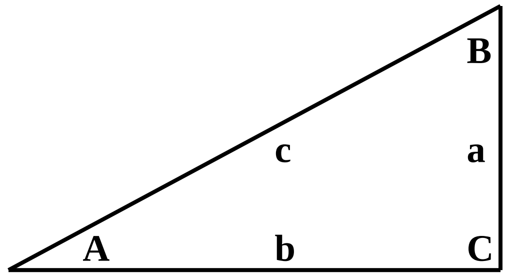
$y = \sqrt{(r+x)\,(r-x)} = \sqrt{r^2 - x^2}$

$x = \sqrt{(r+y)\,(r-y)} = \sqrt{r^2 - y^2}$

$r = \sqrt{x^2 + y^2}$

$vers\ a = (r-x)/r = 1 - x/r = covers\ \theta$
$exsec\ a = (r-x)/x = r/x - 1 = coexsec\ \theta$
$covers\ a = (r-y)/r = 1 - y/r = vers\ \theta$
$coexsec\ a = (r-y)/y = r/y - 1 = exsec\ \theta$

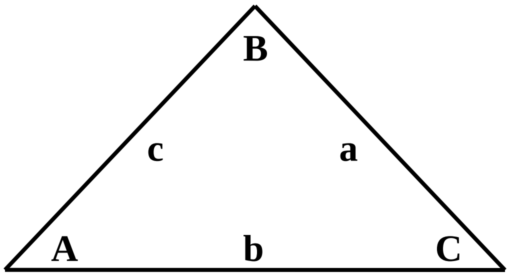
B. Right Triangles



$C = 90°$

Given	To Find	Formulas
a, c	b A B	$\sqrt{c^2 - a^2}$ $\sin A = a/c$ $\cos B = a/c$
	Area	$\frac{a}{2}\sqrt{c^2 - a^2}$
a, b	c A B	$\sqrt{a^2 + b^2}$ $\tan A = a/b; \cot A = b/a$ $\tan B = b/a; \cot B = a/b$
	Area	$\frac{ab}{2}$
A, a	b c B	$a \cot A$ $a/ \sin A$ $90° - A$
	Area	$\frac{a^2 \cot A}{2}$
A, b	a c B	$b \tan A$ $b/\cos A$ $90° - A$
	Area	$\frac{b^2 \tan A}{2}$
A, c	a b B	$c \sin A$ $c/ \cos A$ $90° - A$
	Area	$\frac{c^2(\sin A)(\cos A)}{2} = \frac{\mathrm{c}^2 \sin 2\mathrm{A}}{4}$

C. Oblique Triangles



Law of Sines: $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin B} = \frac{c}{\sin C}$

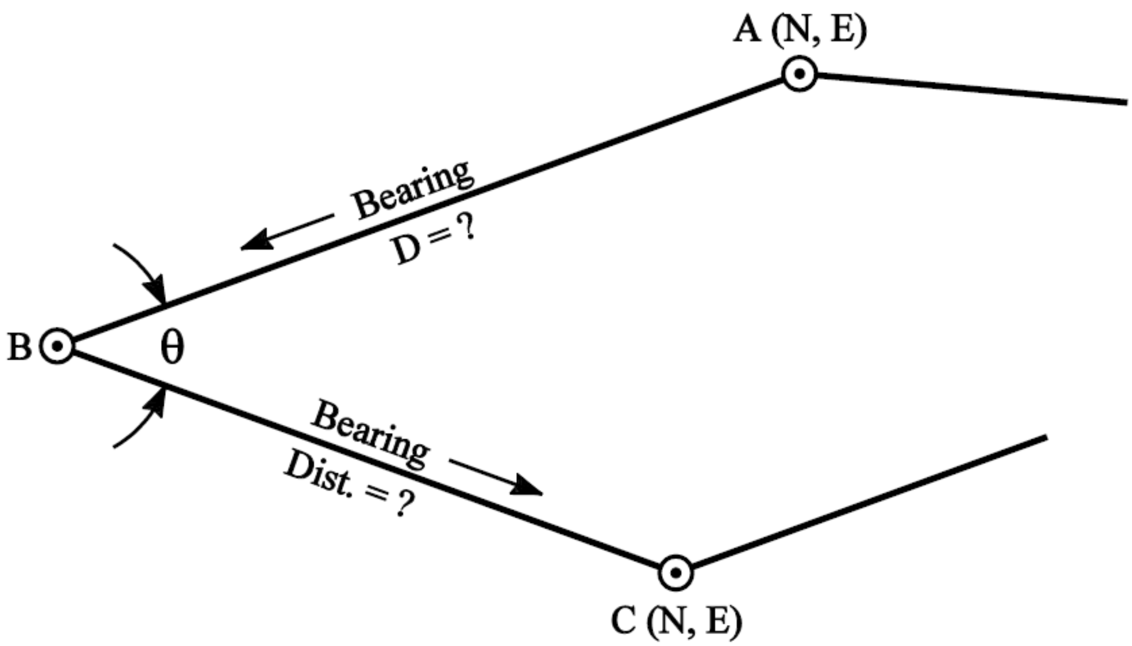
Law of Cosines: $a^2 = b^2 + c^2 - 2\mathrm{bc}\cos \mathrm{A}$

or $\cos A = \frac{b^2+c^2-a^2}{2bc}$

Given	To Find	Formulas
a, b, c	A, B, C Using $s = ½(a + b + c)$	Law of Cosines: $\sin ½A = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\cos ½A = \sqrt{\frac{s(s-a)}{bc}}$ $\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$ *Note: For angles B & C, make appropriate substitutions in these formulas.*
	Area	$\sqrt{s(s-a)(s-b)(s-c)}$
a, A, B	b C c	Law of Sines $180° - (A + B)$ Law of Sines: $\frac{a\sin(A+B)}{\sin A}$
	Area	$\frac{a^2 \sin B \sin(A + B)}{2\sin A}$
a, b, A	B C c	Law of Sines $180° - (A + B)$ Law of Sines: $\frac{a\sin(A+B)}{\sin A}$
a, b, C	c A B	Law of Cosines $\tan A = \frac{a\sin C}{b - (a\cos C)}$ $180° - (A + C)$
	Area	$½ab\sin C$
A, B, C, a	Area	$\frac{a^2(\sin B)(\sin C)}{2\sin A}$

D. Intersection Problems

1. Case I - Adjacent unknown distances



Given:
1. Bearing of lines $\overline{AB}$ & $\overline{BC}$
2. Coordinates of pts A and C
3. ΔN = Difference in Northings = $N_C - N_A$
4. ΔE = Difference in Eastings = $E_C - E_A$

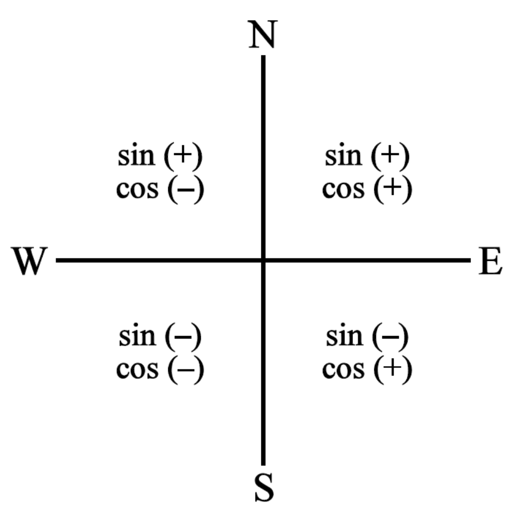
To Find:
1. Distances AB and BC

Formulas:

$$AB = \frac{(\Delta E)(\cos \overline{BC}) - (\Delta N)(\sin \overline{BC})}{\sin \theta}$$

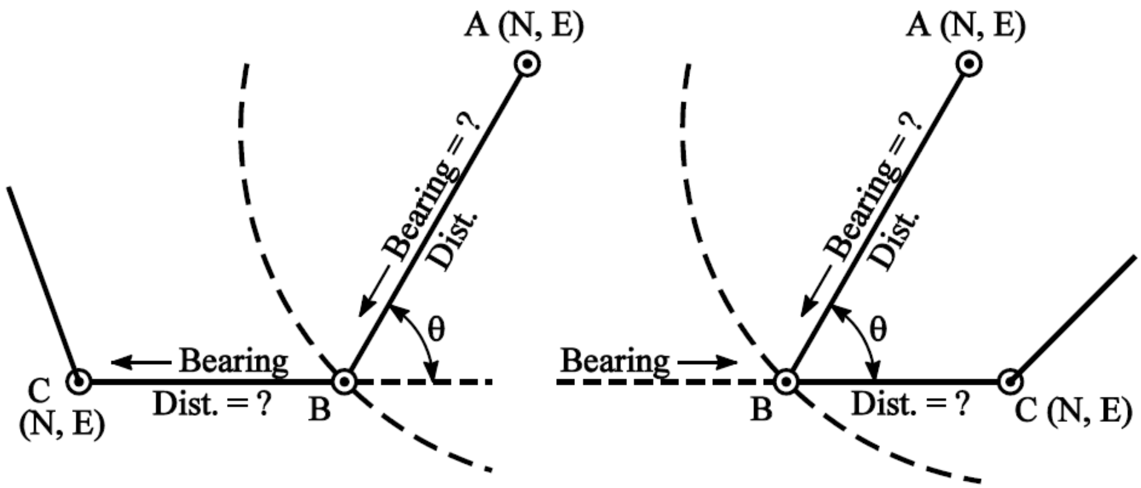
$$BC = \frac{(\Delta E)(\cos \overline{AB}) - (\Delta N)(\sin \overline{AB})}{\sin \theta}$$

NOTE: Observe all algebraic signs of the functions ΔN and ΔE. Signs of the functions are determined by the quadrant of the bearing used in the calculations.



2. Case II - Unknown distance and adjacent unknown bearing

(The intersection of adjacent sides of a traverse or of a curve and a straight line.)



Given:

1. Bearing of line $\overline{BC}$
2. Distance of AB
3. Coordinates of pts A and C
4. ΔN = Differences in Northings = $N_C - N_A$
5. ΔE = Differences in Eastings = $E_C - E_A$

To Find:

1. Bearing of $\overline{AB}$ by solving for θ
2. Distance CB

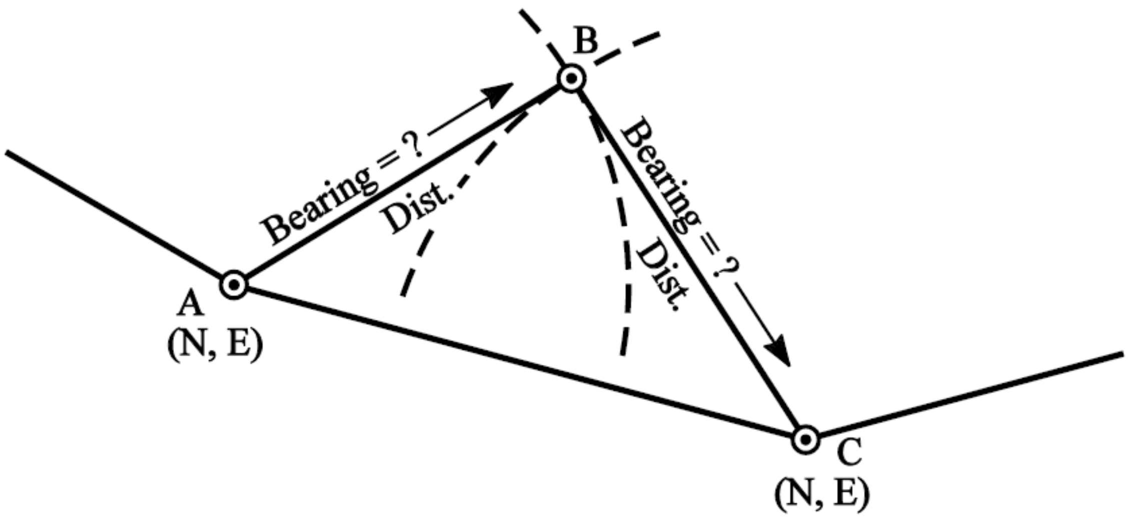
Formula:

$$\sin\theta = \frac{(\Delta E)(\cos\overline{BC}) - (\Delta N)(\sin\overline{BC})}{AB}$$

See note on page C-4.

3. Case III - Two adjacent unknown bearings

(The intersection of two curves or adjacent sides of a traverse.)



Given:
1. Coordinates of pts A and C
2. Distances AB and CB

To Find:
1. Bearings of $\overline{AB}$
2. Bearing of $\overline{CB}$

Solution:
1. Inverse between pts A and C to find distance AC and Bearing AC.
2. With all three sides known, use the Law of Cosines to solve for angle A or angle C or use:

$$\cos \tfrac{1}{2}A = \sqrt{\frac{S(S-CB)}{(AC)(CB)}}$$

$$\cos \tfrac{1}{2}A = \sqrt{\frac{S(S-CB)}{(AC)(CB)}}$$

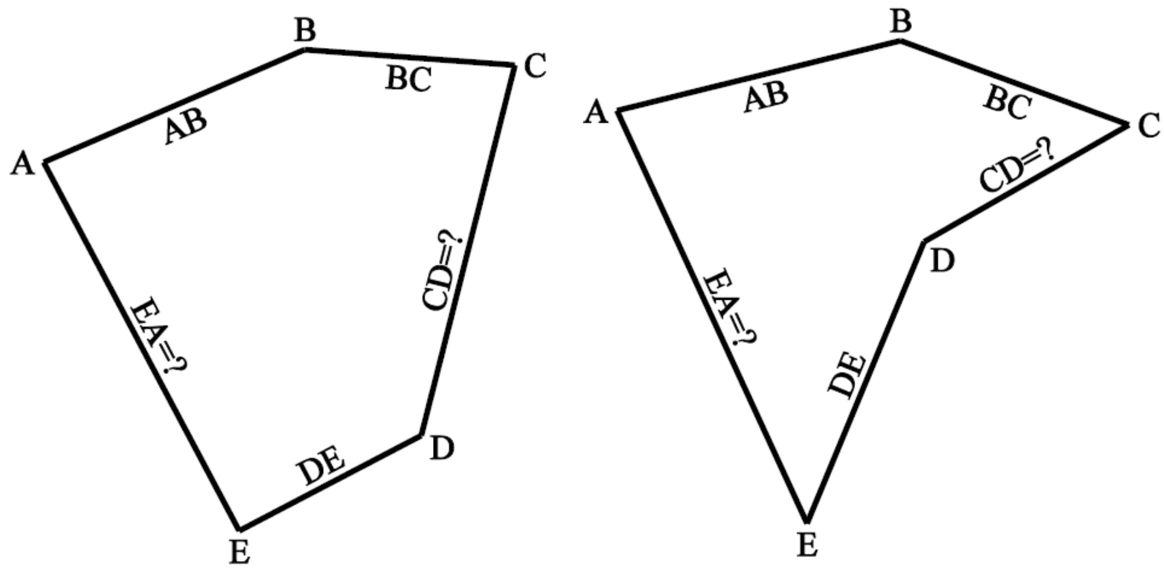
Where $S = \tfrac{1}{2}(AB + CB + AC)$

3. Solve the other angle of the two (A or C) by either the Law of Sines or the Law of Cosines.

$$\sin C = \frac{AB}{CB}\sin A$$

$$\sin A = \frac{CB}{AB}\sin C$$

4. Case IV - Two unknowns, non-adjacent sides of a closed traverse



Since any closed figure is a closed traverse, the positions of the sides can be rearranged to make the two unknown sides adjacent, as shown in the above figures. Make these two sides the last two traverse courses. Solve by:

1. Case I, if both unknowns are distances.
2. Case II, if one unknown is a distance and the other is a bearing.
3. Case III, if both unknowns are bearings.